

SELF-TUNING SOLUTIONS OF THE COSMOLOGICAL CONSTANT *

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I briefly review the cosmological constant problem and attempts toward its solution, and present the first nontrivial example for the self-tuning mechanism with a $1/H^2$ term with the antisymmetric field strength H_{MNPQ} in a 5D RS-II setup.

1 Introduction

It is generally believed that the cosmological constant problem is the most severe hierarchy problem in particle physics.¹ The hierarchy problem has been formulated since 1975 in connection with grand unified theories(GUTs). GUTs introduce two scales which differ by a factor of 10^{14} . At the classical Lagrangian level, there appear parameters which are of order 10^{16} GeV. But loop corrections and spontaneous symmetry breaking enter and after the dust settles down we require to have an electroweak vacuum expectation value(VEV) of order 100 GeV. This implies that the parameter of the Higgs boson mass must satisfy $M_H^2 + \Delta M_H^2 = O(10^{-28})M_{GUT}^2$, which is a fine-tuning problem known as the so-called gauge hierarchy problem. When we consider different scales in the same Lagrangian, in general we encounter this kind of hierarchy problem.

Gravity is described by metric $g_{\mu\nu}$. Then the metric theory of gravity is given by the action

$$\text{Action} = \int d^4x \sqrt{-g} \left(\frac{M^2}{2} R - V_0 + \cdots \right) \quad (1)$$

where V_0 is the vacuum energy. The above action leads to the Einstein equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + 8\pi G V_0 g_{\mu\nu} = 8\pi G T_{\mu\nu}. \quad (2)$$

Actually, Einstein introduced the cosmological constant on the LHS of the above equation as Λ instead of $8\pi G V_0$ in 1917 to obtain a seemingly static universe. But the Hubble expansion observed 12 years later invalidated this argument. Even at the time of Einstein, the cosmological constant problem

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could have been formulated as a hierarchy problem. The parameter M appearing in the action is the Planck mass $M = 2.44 \times 10^{18}$ GeV. If gravity introduces this large mass, then any other parameter in gravity is expected to be of that order, in particular V_0 in Eq. (1). However, the bound on the vacuum energy is very strong $< (0.01 \text{ eV})^4$, which implies a fine-tuning of order 10^{-120} . Thus, this cosmological constant problem is the most severe hierarchy problem.

Usually a hierarchy problem is understood if there exists a symmetry related to it. The difficulty with the cosmological constant problem is that there is no such symmetry working. One obvious symmetry is the scale invariance but it is badly broken by the mass terms, for example. The electroweak scale introduces a mass scale which is about 10^{56} times larger than the observed cosmological constant bound. The cosmological constant problem has surfaced in particle physics for a need to set the minimum point of the Higgs potential. But we cannot find a theoretical guideline where to put the vacuum energy, which is another way of stating the hierarchy problem.² Since then there have been several attempts toward a solution of the problem under the name of probabilistic interpretation,³ boundary of different phases,⁴ wormholes,⁵ anthropic principle,⁶ etc.⁷ Among these the most interesting ones are the probabilistic interpretation and anthropic solution. Note that the probabilistic interpretation is based on the multi-vacua possibility.

The anthropic solution is a working one as far as multi-vacua are allowed in the theory. Probabilistic interpretation is also based on the multi-vacua possibility. It is based on the requirement that life evolution is not very much affected by the existence of the cosmological constant. Galaxy formation may be hindered if the cosmological constant is too large. Weinberg obtained a bound on the vacuum energy density, $\rho < 550\rho_c$, from the condition that condensation of matter occurs. Thus, in the anthropic solution the fine-tuning is reduced to 1 out of 1000.

2 Self-tuning Solutions

2.1 The old version

For a given action, if there exists a flat space solution without a fine-tuning then it is called a self-tuning solution or an undetermined integration constant(UIC) solution in early eighties. Note that in 4D it is not possible to have a flat space solution $ds^2 = d\mathbf{x}^2 - dt^2$ with a nonvanishing Λ . Only de Sitter space($\Lambda > 0$) or anti de Sitter space($\Lambda < 0$) solutions are possible. Therefore, in 4D one needs an extreme fine-tuning to satisfy the observed

Table 1. Several fundamental scales accompanying vacuum energy

(energy density) $^{\frac{1}{4}}$	Physics of interest
10^{18} GeV	Gravity is strong
10^{16} GeV?	Spontaneous breaking of GUT symmetry
10^{10-13} GeV?	PQ symmetry breaking Seed for SUSY breaking
100 GeV	Electroweak symmetry breaking
1 GeV – 100 MeV	QCD chiral symmetry breaking
10^{-3} eV	Present vacuum energy

bound on the vacuum energy.

But suppose that there exists an UIC. Witten,⁴ using the earlier idea in the 11d supergravity,⁸ showed this possibility with a four index antisymmetric field strength $H_{\mu\nu\rho\sigma}$. In 4D, it is not a dynamical field but equation of motion for $H_{\mu\nu\rho\sigma}$ can lead to a constant contribution to the vacuum energy $\sim c^2$ via a nonvanishing VEV $H_{\mu\nu\rho\sigma} = c\epsilon_{\mu\nu\rho\sigma}$. This UIC c can be adjusted so that the final cosmological constant vanishes. Once c is determined there is no other parameter to adjust to cancel further additions of the cosmological constant. Namely, at different stages of the spontaneous symmetry breaking(as shown in Table 1) we do not have enough UIC. This example did not work because $H_{\mu\nu\rho\sigma}$ is not a dynamical field. Also, Hawking³ introduced the four index antisymmetric field strength to explain his probabilistic choice of vanishing cosmological constant in a multi universe scenario. Again in 4D it is not a dynamical field and there are not enough UIC.

In these old versions for the self-tuning solution, one *does not require that only the flat space solution is the solution of the equation of motion*. They allowed the de Sitter and anti de Sitter space solutions. But the flat space is chosen from the other principles.^{3,4}

2.2 The new version

Recently, a more restricted class of self-tuning solutions is suggested for a solution of the cosmological constant problem.⁹ It requires the existence of a flat space solution but forbid de Sitter and anti de Sitter space solutions. It is a very fascinating idea, presumably dreamed of for a long time. But the recent interest came from the 5D brane scenario of Randall-Sundrum(RS).¹⁰ The idea can be seen to be plausible by observing that RS models can allow flat space solutions even starting with nonvanishing brane tension and negative bulk cosmological constant. However, toward the flat space solutions one should have fine-tuned the parameters. Therefore, the RS models can be

a playground toward an effective 4D flat spacetime. In this spirit, the new version tried to obtain the flat space solutions without fine-tuning but without de Sitter and anti de Sitter space solutions.⁹ However, their model introduced an essential singularity and a proper treatment of this singularity reintroduced a fine-tuning condition,¹¹ and hence it is fair to say that there has not appeared a working self-tuning model in the new version.

Since the rise and fall of this new version is interesting toward future discovery of a self-tuning solution, I will briefly review it.

The RS-II model¹⁰ is an alternative to compactification. With the bulk anti de Sitter space(AdS), the localization of gravity makes it acceptable to have an uncompactified extra dimension. A brane is located at $y = 0$, where matter fields are assumed to live. Then the 5D Lagrangian is

$$\mathcal{L} = \frac{M^3}{2}(R - \Lambda_b) + (\mathcal{L}_{matter} - \Lambda_1)\delta(y) \quad (3)$$

where M is a 5D Planck mass. One is interested in a 4D-flat solution,

$$ds^2 = \beta(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2. \quad (4)$$

With a Z_2 symmetry, one can find a solution of (4), $\beta(y) = \beta_0 \exp(-k|y|)$ where $k = (-\Lambda_b/6M^3)^{1/2}$. The boundary condition at $y = 0$ dictates a fine-tuning between k and $k_1 = \Lambda_1/6$, $k = k_1$, where we set $M = 1$. If we put more branes, then there are more conditions to satisfy due to the freedom to introduce more brane tension parameters Λ_i at brane B_i . Thus, RS-II model is the easiest one to try for a self-tuning solution.

First try: Kachru et al. and Arkani-Hamed et al.⁹ tried the following 5D Lagrangian with a brane at $y = 0$,

$$\mathcal{L} = R - \Lambda e^{a\phi} - \frac{4}{3}(\nabla\phi)^2 - V e^{b\phi}\delta(y) \quad (5)$$

where the fundamental scale $M/2$ is set to 1. It is a RS-II type model with a massless scalar field ϕ in the bulk. This scalar interacts with the brane tension through the V term. The relevant equations are

$$\begin{aligned} \text{dilataton : } & \frac{8}{3}\phi'' + \frac{32}{3}A'\phi' - a\Lambda e^{a\phi} - bV\delta(y)e^{b\phi} = 0 \\ (55) : & 6(A')^2 - \frac{2}{3}(\phi')^2 + \frac{1}{2}\Lambda e^{a\phi} = 0 \\ (55), (\mu\nu) : & 3A'' + \frac{4}{3}(\phi')^2 + \frac{1}{2}e^{b\phi}V\delta(y) \end{aligned} \quad (6)$$

The 4D flat ansatz $A(y) = \ln \beta(y)^{1/2}$ may imply a zero cosmological constant if there exists a solution of these equations. Indeed, they found a bulk

solution $\phi = \pm(3/4) \ln |(4/3)y + c| + d$, with $A' = \pm(1/3)\phi'$ and $\Lambda = 0$. Here, c and d are integration constants and there remains only one integration constant after satisfying the boundary condition at $y = 0$. There are two solutions, one without a singularity and the other with a singularity at $y_c = -(3/4)c$. The nonsingular solution diverges logarithmically at large $|y|$ and localization of gravity near the brane is not realized. The singular solution has a naked singularity at y_c . An effective 4D theory is obtained by integrating out with y , and hence we cannot ignore the space $y \geq y_c$. Even if the bulk for a given y is flat, the effective theory must know the whole y space. If we cut off the integration at $y = y_c - \epsilon$, then certainly the resultant 4D cosmological constant would not be zero, which is obvious since the singular point is an essential singularity. So the solution is incomplete with the given flat space ansatz.

Second try: Förste et al.¹¹ cured the singularity problem of the above example, by inserting a brane at $y = y_c$. They showed that there exists a solution but with one fine-tuning between parameters. Therefore, we have not obtained a new type self-tuning solution yet.

3 A Self-tuning Solution with $1/H^2$

A naive try is to introduce a dynamical spin-0 field whose mass is zero so that it affects the whole region of the bulk. Kachru et al.⁹ in fact attempted to use a massless scalar. In this case one must assume the form of the potential. However, if a massless scalar such as a Goldstone boson arises from the symmetry in the theory, it will be much better. It is achieved by an antisymmetric tensor field. For one dynamical degree, the rank is $(2 + n)$ in $(4 + n)$ -dimensional spacetime. In 5D RS model, the antisymmetric tensor field is $A_{MNP}(M, N, P = 0, 1, 2, 3, 5)$ whose four-form field strength is H_{MNPQ} . The four form field is invariant under the gauge transformation,

$$A_{MNP} \rightarrow A_{MNP} + \partial_{[M} \lambda_{NP]}. \quad (7)$$

There will be a $U(1)$ gauge symmetry remaining with one massless pseudoscalar field. The massless one is a : $\partial_M a = \sqrt{-g} \epsilon_{MNPQR} H^{NPQR} / 4!$. Toward a flat-space solution we adopt the following ansatz:

$$\text{Ansatz 1} = (\text{Flat } 4D), \quad \text{Ansatz 2} = (D = 4 \text{ chosen}) \quad (8)$$

where μ, ν, \dots are the 4D indices.

(i) With H^2 term

The simplest term H^2 does not lead to a self-tuning solution. We use,

$$\begin{aligned} \text{Ansatz 1 (Flat 4D)} : ds^2 &= \beta(y)^2 \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \\ \text{Ansatz 2 (D = 4 chosen)} : H_{\mu\nu\rho\sigma} &= \epsilon_{\mu\nu\rho\sigma} \frac{1}{\sqrt{-g}n(y)} \end{aligned}$$

The field equation of H , (55) and $(\mu\nu)$ component Einstein equations lead to the following solutions

$$\begin{aligned} \Lambda_b < 0 : (a/k)^{1/4} [\sinh(4k|y| + c)]^{1/4} \\ \Lambda_b > 0 : (a/k)^{1/4} [\sin(4k|y| + c')]^{1/4} \\ \Lambda_b = 0 : (|4a|y| + c'')^{1/4} \end{aligned}$$

For a localizable metric at $y = 0$, there exists a singularity at $-c/4k$, etc. Thus, another brane is necessary and we need a fine-tuning as in the Kachru et al case.⁹

(ii) With $1/H^2$ term

But we find that there exists a solution with $1/H^2$ term. Using the unit $M = 1$, we take the following action,

$$S = \int d^4x \int dy \sqrt{-g} \left\{ \frac{1}{2} R - \frac{2 \cdot 4!}{H_{MNPQ} H^{MNPQ}} - \Lambda_b - \Lambda_1 \delta(y) \right\} \quad (9)$$

As before we use the following ansatz,

$$\begin{aligned} \text{Ansatz 1 (Flat 4D)} : ds^2 &= \beta(y)^2 \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \\ \text{Ansatz 2 (D = 4 chosen)} : H_{\mu\nu\rho\sigma} &= \epsilon_{\mu\nu\rho\sigma} \sqrt{-g} \frac{1}{n(y)}. \end{aligned}$$

The H field equation is, $\partial_M [\sqrt{-g} H^{MNPQ} / H^4] = \partial_\mu [\sqrt{-g} H^{\mu NPQ} / H^4] = 0$. Therefore, n is a function of y only. The (55) component Einstein equation is $6(\beta'/\beta)^2 = -\Lambda_b - (\beta^8/A)$, implying a solution in the anti de Sitter bulk $\Lambda_b < 0$. Here, the integration constant A is positive and $2n^2 = \beta^8/A$. The $(\mu\mu)$ equation is $3(\beta'/\beta)^2 + 3(\beta''/\beta) = -\Lambda_b - \Lambda_1 \delta(y) - 3(\beta^8/A)$. We impose a Z_2 symmetry. The boundary condition at $y = 0$ is $[\beta'/\beta]_{0+} = -\Lambda_1/6$. Then, the flat solution is found to be

$$\beta(|y|) = [(a/k) \cosh(4k|y| + c)]^{-1/4} \quad (10)$$

where $k = \sqrt{-\Lambda_b/6}$, $a = \sqrt{1/6A}$, and $k_1 = \Lambda_1/6$. The bulk solution (10) contains integration constants a and c . Here, a is basically the charge of the

universe and is determined by the definition of 4D Planck mass. But, c is undetermined and fixed by the boundary condition at $y = 0$,

$$\tanh c = \frac{k_1}{k} = \frac{\Lambda_1}{\sqrt{-6\Lambda_b}}. \quad (11)$$

Because of the limited range for \tanh , the solution is possible in the region, $|\Lambda_1| < \sqrt{-\Lambda_b/6}$. This relation shows that it is possible to have a flat space solution for any value of the brane tension Λ_1 with anti de Sitter space bulk. In this sense it is an old type self-tuning solution, i.e. there exists a flat solution for any parameter value within a finite range. If the observable sector VEV (i.e. Λ_1) changes, the solution adjusts so that c matches the condition (11). If there were no dynamical field, this adjustment would be impossible, but in our case H has one dynamical field and it is possible to change the shape of the solution according to a changing Λ_1 .

Note that $\beta(|y|)$ is a decreasing function of $|y|$ and it tends to 0 as $y \rightarrow \infty$. This property seems to be needed for a self-tuning solution. The key points of our solution are:

- **(1) β has no naked singularity:** If $\beta(y)$ has a singularity at $y = y_c$, we should cutoff the space up to y_c . In this case, we would not obtain a vanishing 4D cosmological constant. To avoid the singularity, we should introduce another brane at $y = y_c$, and there results a fine-tuning since introduction of an additional brane introduces another brane tension which cannot be an arbitrary parameter to give a flat space. Our solution does not have a singularity in the whole y space.
- **(2) Finite 4D Planck mass:** Integrating with respect to y , we obtain a finite 4D Planck mass.¹²
- **(3) Self-tuning solution:** We can show explicitly that by integrating out with y the resultant 4D cosmological constant is zero. In this consideration, the surface term gives an important contribution.¹²

So far we considered the time-independent solutions. The next simplest solutions are the de Sitter and anti de Sitter type time dependent solutions. These are parametrized by the curvature λ (+ for de Sitter and - for anti de Sitter) with the metric $g_{\mu\nu} = \text{diag.}(-1, e^{2\sqrt{\lambda}t}, e^{2\sqrt{\lambda}t}, e^{2\sqrt{\lambda}t})$ for dS₄ and $g_{\mu\nu} = \text{diag.}(-e^{2\sqrt{-\lambda}x_3}, e^{2\sqrt{-\lambda}x_3}, e^{2\sqrt{-\lambda}x_3}, 1)$ for AdS₄. The 4D Riemann tensor becomes $R_{\mu\nu} = 3\lambda g_{\mu\nu}$ with $g_{\mu\nu}$ given above. One particularly interesting equation is the (55) component equation

$$(\beta')^2 = \lambda - \frac{\lambda_b}{6}\beta^2 - \frac{\beta^{10}}{6A}. \quad (12)$$

The cosmological constant obtained from these equations is λ . In fact, in the RS-II model one can explicitly show this,¹² using the exact solution given in Ref. [11]. In our case, we have not obtained an exact time dependent solution, but can show the existence of de Sitter and anti de Sitter space solutions theoretically and numerically.¹² For example the de Sitter space solution is possible since Eq. (12) allows a point y_h (the de Sitter space horizon) where $\beta' \neq 0$ for $\beta = 0$. For the de Sitter space solution, for example, integration from $-y_h$ to $+y_h$ should give the vacuum energy λ as in the RS-II case.¹² On the other hand, the AdS₄ solution does not give a localized gravity.

4 Conclusion and Comments

We have obtained an old type self-tuning solution in the RS setup using $1/H^2$ term: (i) from 5D, 4D is chosen by $\langle H_{\mu\nu\rho\sigma} \rangle$, (ii) there is no tachyonic Kaluza-Klein states, (iii) for some finite range of the brane tension there always exists a flat space solution, but (iv) we have not obtained an exact time dependent solution.

Toward a solution of the cosmological constant problem, however, we note two essential points encountered in our solution:

- We adopted a peculiar kinetic energy term, $1/H^2$. In this sense the solution can be considered as another fine-tuning. However, we have shown that the flat space solution is from more general type of the action $1/H^{2n} (n > 0)$,¹² and hope that these forms will be understood in the future. A more immediate question is whether it gives consistent physics at low energy. One can consider a consistent field theory model from $\mathcal{L} = -(1/8)H^2(F^2)^2 - (1/4)F^2$ where F is a $U(1)$ field strength $F_{\mu\nu}$, i.e. $F^2 = F_{\mu\nu}F^{\mu\nu}$. The Gaussian integral would choose $F^2 = -1/H^2$ and after integrating out the $U(1)$ field we obtain $1/H^2$ term. Therefore, the consideration of $1/H^2$ as an effective interaction below some energy scale makes sense. Since H corresponds to an exactly massless boson, it can be considered as a Goldstone boson. Thus, we may construct a theory for a massless Goldstone boson (\neq dilaton) toward a self-tuning solution.
- In our case, we observed that there exist de Sitter and anti de Sitter space solutions also. This arose a question: Why the flat space? In the literature, there already exists a proposal: the probabilistic choice by Hawking.³ The probability to choose the flat space with $\Lambda = 0^+$ is exponentially larger than the other cases. For this idea to work, the vacua allows multi universes and it is so in our example. Duff's point¹⁴ on the Hawking solution seems to have neglected the surface contribution. In

our example, a more concrete scenario would be possible if we obtain a time dependent solution transforming one with a Λ_1 to another with a different Λ_1 .

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